Axioms and theorems of Boolean algebra

revision

- identity
  1. $X + 0 = X$ 1D. $X \cdot 1 = X$
- null
  2. $X + 1 = 1$ 2D. $X \cdot 0 = 0$
- idempotency:
  3. $X + X = X$ 3D. $X \cdot X = X$
- involution:
  4. $(X')' = X$
- complementarity:
  5. $X + X' = 1$ 5D. $X \cdot X' = 0$
- commutativity:
  6. $X + Y = Y + X$ 6D. $X \cdot Y = Y \cdot X$
- associativity:
  7. $(X + Y) + Z = X + (Y + Z)$ 7D. $(X \cdot Y) \cdot Z = X \cdot (Y \cdot Z)$

Axioms and theorems of Boolean algebra (cont'd)

revision

- distributivity:
  - 8. $X \bullet (Y + Z) = (X \bullet Y) + (X \bullet Z)$ 8D. $X + (Y \bullet Z) = (X + Y) \bullet (X + Z)$
- uniting:
  - 9. $X \bullet Y + X \bullet Y' = X$ 9D. $(X + Y) \bullet (X + Y') = X$
- absorption:
  - 10. $X + X \bullet Y = X$ 10D. $X \bullet (X + Y) = X$
  - 11. $(X + Y') \bullet Y = X \bullet Y$ 11D. $(X \bullet Y') + Y = X + Y$
- factoring:
  - 12. $(X + Y) \bullet (X' + Z) = X \bullet Z + X' \bullet Y$ 12D. $X \bullet Y + X' \bullet Z = (X + Z) \bullet (X' + Y)$
- concensus:
  - 13. $(X \bullet Y) + (Y \bullet Z) + (X' \bullet Z) = X \bullet Y + X' \bullet Z$ 13D. $(X + Y) \bullet (Y + Z) \bullet (X' + Z) = (X + Y) \bullet (X' + Z)$

Axioms and theorems of Boolean algebra (cont'd)

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- de Morgan's:
  - 14. (X + Y + ...)' = X' • Y' • ... 14D. (X • Y • ...)' = X' + Y' + ...
- generalized de Morgan's:
  - 15. $f'(X_1,X_2,...,X_n,0,1,+,\bullet) = f(X_1',X_2',...,X_n',1,0,\bullet,+)$
- establishes relationship between • and +

Axioms and theorems of Boolean algebra (cont'd)

- Duality
  - a dual of a Boolean expression is derived by replacing
    - • by +, + by •, 0 by 1, and 1 by 0, and leaving variables unchanged
  - any theorem that can be proven is thus also proven for its dual!
  - a meta-theorem (a theorem about theorems)
- duality:
  16. X + Y + ... ⇔ X • Y • ...
- generalized duality:
  17. f (X_1,X_2,...,X_n,0,1,+,•) ⇔ f(X_1,X_2,...,X_n,1,0,•,+)

- Different than deMorgan's Law
  - this is a statement about theorems
  - this is not a way to manipulate (re-write) expressions

Activity

- Prove the following using the laws of Boolean algebra:
  - $(X \bullet Y) + (Y \bullet Z) + (X' \bullet Z) = X \bullet Y + X' \bullet Z$