Chapter 4 – Impedance Matching

1 – Quarter-wave transformer, series section transformer
2 – Stub matching, lumped element networks, feed point location
3 – Gamma match
4 – Delta- and T-match, Baluns
1 – 2-port network
2 – Smith Chart

REVISION
2-Port Network Representation

- Six ways to represent a two-port network in terms of $V$ and $I$ at each port:
  - Z-matrix: open-circuit impedance
  - Y-matrix: short-circuit admittance
  - ABCD-matrix: chain or transmission parameters
  - B-matrix: inverse transmission parameters
  - H-matrix: hybrid parameters
  - G-matrix: inverse hybrid parameters
2 -Port Network Representation

- 7th way to represent a two-port network in terms of waves entering and leaving each port:
  - S-matrix scattering parameters

- In general, there could be n ports in a network:
**[S] for Two-Port Network**

Definition of normalized voltage waves:

*Incident wave*: \( a_1 = \frac{V_{i1}}{\sqrt{Z_{o1}}} = \sqrt{P_{i1}} \quad a_2 = \frac{V_{i2}}{\sqrt{Z_{o2}}} = \sqrt{P_{i2}} \)

*Reflected wave*: \( b_1 = \frac{V_{r1}}{\sqrt{Z_{o1}}} = \sqrt{P_{r1}} \quad b_2 = \frac{V_{r2}}{\sqrt{Z_{o2}}} = \sqrt{P_{r2}} \)

Scattering parameters are then defined as:

\[
\begin{align*}
  b_1 &= S_{11}a_1 + S_{12}a_2 \\
  b_2 &= S_{21}a_1 + S_{22}a_2
\end{align*}
\]

or

\[
\begin{bmatrix}
  b_1 \\
  b_2
\end{bmatrix} =
\begin{bmatrix}
  S_{11} & S_{12} \\
  S_{21} & S_{22}
\end{bmatrix}
\begin{bmatrix}
  a_1 \\
  a_2
\end{bmatrix}
\]

or

\[
[b] = [S][a]
\]
**S\textsubscript{11} for a One-Port Network**

Measured S\textsubscript{11} of an 800 MHz square microstrip patch antenna on a 1.6 mm FR4 substrate.
$S_{21}$ for a One-Port Network

Measured $S_{21}$ of a 542 MHz hair-pin bandpass filter on a 1.6 mm FR4 substrate
Smith Chart Impedance or Admittance
Smith Chart: Exercises

1. Use as a Z-chart, locate s/c, o/c, Zo , 1+ j1
2. Use as a Y-chart, locate s/c, o/c, Yo , 1+ j1
3. Move towards generator l/l
4. Move towards load l/l
5. Read VSWR, |G |, Vmax , Vmin
6. Read attenuation
1 – Introduction
2 – Importance of Impedance Matching
INTRODUCTION

• The operation of an antenna system over a frequency range is not completely dependent upon the frequency response of the antenna element itself but rather on the frequency characteristics of the transmission line-antenna element combination.

• In practice, the characteristic impedance of the transmission line is usually real whereas that of the antenna element is complex.

• Also the variation of each as a function of frequency is not the same.

• The efficient coupling-matching networks must be designed which attempt to couple-match the characteristics of the two devices over the desired frequency range.
Feed-Point Impedance: $Z_a$

- $Z_a = \text{antenna impedance at its feed-point.}$

- $Z_a$ is complex generally.

\[ Z_a = R_a + jX_a \]
Importance of Impedance Matching

• Increased power throughput (e.g. maximum power transfer)
• Increased power handling capability in a transmission line (due to reduced VSWR)
• Reduced effects on impedance matching sensitive circuits (e.g. frequency pulling effect on the signal source)
• With "controlled mismatch", an RF amplifier can operate with minimum noise generation (i.e. minimum noise figure)
Concept of maximum power transfer

In lump circuit

\[ P_L = \frac{1}{2} V_L I = \frac{1}{2} I^2 Z_L = \frac{1}{2} \left( \frac{V_i}{Z_L + Z_o} \right)^2 Z_L \]

Power deliver at \( Z_L \) is

Power maximum whence \( Z_L = Z_o \)
continue

In transmission line

The important parameter is reflection coefficient \( \rho = \frac{Z_L - Z_o}{Z_L + Z_o} \)

No reflection whence \( Z_L = Z_o \), hence \( \rho = 0 \)

The load \( Z_L \) can be matched as long as \( Z_L \) not equal to zero (short-circuit) or infinity (open-circuit)
Features of Impedance Matching Networks

- **Complexity**
  - use the simplest --> cheap, low loss

- **Bandwidth**
  - perfect match usually at a single frequency
  - wider bandwidth increases complexity

- **Implementation**
  - select matching components to suit an application:
    - lumped-element (L, C, R)
    - distributed-element (transmission line, waveguides)

- **Adjustability**
  - for loads with variable impedance
MATCHING METHODS

1 – calculations
2 – smith chart
Matching with lumped elements

The simplest matching network is an L-section using two reactive elements.

Configuration 1
Whence $R_L > Z_0$

Configuration 2
Whence $R_L < Z_0$

\[ Z_L = R_L + jX_L \]
If the load impedance (normalized) lies in unity circle, configuration 1 is used. Otherwise configuration 2 is used.

The reactive elements are either inductors or capacitors. So there are 8 possibilities for matching circuit for various load impedances. Matching by lumped elements are possible for frequency below 1 GHz or for higher frequency in integrated circuit (MIC, MEM).
Impedances for serial lumped elements

Serial circuit

Reactance | relationship | values
--- | --- | ---
+ve | $X = 2\pi fL$ | $L = X/(2\pi f)$
-ve | $X = 1/(2\pi fC)$ | $C = 1/(2\pi fX)$
Impedances for parallel lumped elements

Parallel circuit

<table>
<thead>
<tr>
<th>Susceptance</th>
<th>relationship</th>
<th>values</th>
</tr>
</thead>
<tbody>
<tr>
<td>+ve</td>
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</tr>
<tr>
<td>-ve</td>
<td>( B = 1/(2\pi f L) )</td>
<td>( L = 1/(2\pi f B) )</td>
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</table>
Lumped elements for microwave integrated circuit

- Planar resistor
- Chip resistor
- Loop inductor
- Spiral inductor
- Interdigital gap capacitor
- Metal-insulator-metal capacitor
- Chip capacitor

Lossy film
Matching by calculation for configuration 1

For matching, the total impedance of L-section plus $Z_L$ should equal to $Z_o$, thus

$$Z_o = jX + \frac{1}{jB + 1/(R_L + jX_L)}$$

Rearranging and separating into real and imaginary parts gives us

$$B(XR_L - X_L Z_o) = R_L - Z_o \quad *$$

$$X(1 - BX_L) = BZ_o R_L - X_L \quad **$$
continue

Solving for $X$ from simultaneous equations (*) and (**) and substitute $X$ in (**) for $B$, we obtain

$$B = \frac{X_L \pm \sqrt{\frac{R_L}{Z_o}} \sqrt{\frac{R_L^2}{R_L^2} + X_L^2 - Z_o R_L}}{R_L^2 + X_L^2}$$

Since $R_L > Z_o$, then argument of the second root is always positive, the series reactance can be found as

$$X = \frac{1}{B} + \frac{X_L Z_o}{R_L} - \frac{Z_o}{BR_L}$$

Note that two solution for $B$ are possible either positive or negative
Matching by calculation for configuration 2

For matching, the total impedance of L-section plus $Z_L$ should equal to $1/Z_o$, thus

$$\frac{1}{Z_o} = jB + \frac{1}{R_L + j(X + X_L)}$$

Rearranging and separating into real and imaginary parts gives us

$$BZ_o(X + X_L) = Z_o - R_L \quad \text{**}$$

$$\left( X + X_L \right) = BZ_oR_L \quad \text{**}$$
continue

Solving for $X$ and $B$ from simultaneous equations (*) and (**), we obtain

\[
X = \pm \sqrt{R_L(Z_o - R_L)} - X_L
\]

\[
B = \pm \frac{\sqrt{(Z_o - R_L)/R_L}}{Z_o}
\]

Since $R_L < Z_o$, the argument of the square roots are always positive, again two solution for $X$ and $B$ are possible either positive or negative.

+ve inductor -ve capacitor
+ve capacitor -ve inductor
Matching using lumped components
Example

Design an L-section matching network to match a series RC load with an impedance $Z_L = 200 - j100$ W, to a 100 W line, at a frequency of 500 MHz.

Solution

Normalized $Z_L$ we have: $Z_L = 2 - j1$

Parallel L (-j0.7)
Serial C (-j1.2)
Serial L (j1.2)
Parallel C (+j0.3)

Solution 1
Solution 2
continue

\[
C = \frac{b}{2\pi f Z_o} = 0.92 \text{pF}
\]
\[
L = \frac{x Z_o}{2\pi f} = 38.8 \text{nH}
\]

\[
C = \frac{-1}{2\pi f x Z_o} = 2.61 \text{pF}
\]
\[
L = \frac{-Z_o}{2\pi f b} = 46.1 \text{nH}
\]

Solution 2 seems to be better matched at higher frequency
MATCHING TECHNIQUES

1 – Discrete lumped-element
2 – Transmission Line
Impedances for serial lumped elements

Serial circuit

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Impedances for parallel lumped elements

Parallel circuit

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<td>C=B/(2πf)</td>
</tr>
<tr>
<td>-ve</td>
<td>B=1/(2πfL)</td>
<td>L=1/(2πfB)</td>
</tr>
</tbody>
</table>

![Parallel circuit diagram]
Lumped elements for microwave integrated circuit

- Planar resistor
- Chip resistor
- Loop inductor
- Spiral inductor
- Interdigital gap capacitor
- Metal-insulator-metal capacitor
- Chip capacitor

Lossy film

ε

ε_r
Resistive L-Section (Using Resistors)

- Advantage: Broadband
- Disadvantage: Very lossy

Matched conditions:

(a) \( R_{o1} = R_1 + \left( R_2 \parallel R_{o2} \right) \)

(b) \( R_{o2} = R_2 \parallel (R_1 + R_{o1}) \)

Solving (a) and (b) to give \( R_1 \) and \( R_2 \):

\[
R_1 = \sqrt{R_{o1}(R_{o1} - R_{o2})}
\]

\[
R_2 = R_{o2} \sqrt{\frac{R_{o1}}{R_{o1} - R_{o2}}}
\]

\( R_{o1} > R_{o2} \)
Resistive L-Section
(Using Resistors)

- Advantage: Broadband
- Disadvantage: Very lossy

**Attenuation:**

\[
\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{R_2 \parallel R_{o2}}{R_{o1} + R_1 + (R_2 \parallel R_{o2})}
\]

\[R_{o1} > R_{o2}\]
Resistive L-Section: Example

Design a broadband resistive L-section matching network to match a 75 Ω TV antenna output to a 50 Ω transmission line. Calculate the attenuation of the matching network.

Solution:

\[ R_{o1} = 75 \, \Omega, \quad R_{o2} = 50 \, \Omega \]

Substituting these values into previous equations, we get:

\[ R_1 = 43.3 \, \Omega, \quad R_2 = 86.6 \, \Omega \]

\[ \frac{V_{out}}{V_{in}} = \frac{31.7}{75 + 43.3 + 31.7} = 0.2113 = -13.5 \, \text{dB} \]
Reactive L-Section (Using L & C)

• **Advantages:** Low loss, simplicity in design
• **Disadvantages:** Narrow-band, fixed Q

**Matched conditions:**

(a) \[ R_s = jX_s + (jX_p // R_p) \]

(b) \[ R_p = jX_p // (R_s + jX_s) \]

Solving (a) and (b) to give \( X_s \) and \( X_p \):

\[ Q = \sqrt{\frac{R_p}{R_s} - 1} \]

\[ X_s = Q.R_s = R_s \sqrt{\frac{R_p}{R_s} - 1} \]

\[ X_p = \frac{R_p}{Q} = \frac{R_p}{\sqrt{\frac{R_p}{R_s} - 1}} \]

\( X_s \) & \( X_p \): opposite sign

\( R_p > R_s \)

\( R_p > R_s \)

\( X_s \) & \( X_p \): opposite sign

\( X_s = \) +ve for L, -ve for C

\( X_p = \) -ve for C, +ve for L
Reactive LC L-Section: Example

Design an L-section matching network with L and C to match a 50 $\Omega$ source to a 600 $\Omega$ load for maximum power transfer at 400 MHz. Give two solutions.

Solution:

$R_s = 50 \, \Omega$, $R_p = 600 \, \Omega$

Substituting these values into previous equations, we get:

$$Q = \sqrt{\frac{R_p}{R_s}} - 1 = \sqrt{\frac{600}{50}} - 1 = \sqrt{11} = 3.317$$

$$X_s = Q \cdot R_s = 166 \, \Omega$$

$$X_p = \frac{R_p}{Q} = 181 \, \Omega$$
Reactive LC L-Section: Example (cont'd)

**Solution 1:**

\[ X_s = +166 \, \Omega \text{ (inductive)}, \quad X_p = -181 \, \Omega \text{ (capacitive)} \]

\[ L_s = \frac{X_s}{2\pi f} = \frac{166}{2\pi \times 400 \times 10^6} = 66 \, nH \]

\[ C_p = \frac{1}{2\pi f|X_p|} = \frac{1}{2\pi \times 400 \times 10^6 \times 181} = 2.2 \, pF \]

**Solution 2:**

\[ X_s = -166 \, \Omega \text{ (capacitive)}, \quad X_p = +181 \, \Omega \text{ (inductive)} \]

\[ L_p = \frac{X_p}{2\pi f} = \frac{181}{2\pi \times 400 \times 10^6} = 72 \, nH \]

\[ C_s = \frac{1}{2\pi f|X_s|} = \frac{1}{2\pi \times 400 \times 10^6 \times 166} = 2.4 \, pF \]
Reactive LC L-Section: Example (cont'd)

**Solution 1:**

\[ X_s = +166 \, \Omega \text{ (inductive)}, \quad X_p = -181 \, \Omega \text{ (capacitive)} \]

**Solution 2:**

\[ X_s = -166 \, \Omega \text{ (capacitive)}, \quad X_p = +181 \, \Omega \text{ (inductive)} \]
L-Section for Complex Impedances

- If \( R_s \) has a reactance \( jX' \), simply introduce a series with an equal but opposite-sign reactance (\(-jX'\)).

- The \(-jX'\) reactance is then combined with \( jX_s \) to give its final value.

- Similar treatment can be applied to \( R_p \) by introducing a parallel \( jX' \) and a parallel \(-jX'\).
T-Section

- **Advantages:**
  - $R_1$ can be $<$ or $>$ $R_2$
  - Variable Q (higher than L-section)

- **Disadvantages:**
  - More L C elements

**Method:**
(a) Introduce a hypothetical resistance level $R_h$ at $jX_2$, such that $R_h > R_1$ and $R_h > R_2$
   ($R_h$ determines the Q of the matching network)
(b) Split $jX_2$ into $jX_2'$ and $jX_2''$ (in parallel)
(c) Treat the T-section as two L-sections (L-1 and L-2)

**NOTE:** Q of L-1 section $>$ Q of 2-element L-section (as $R_h > R_2$)
T-Section: Q Values

Using the resistance values of the previous 2-element L-section:

(a) Q of original L-section:

\[ Q_L = \sqrt{\frac{R_2}{R_1}} - 1 = \sqrt{\frac{600}{50}} - 1 = 3.317 \]

(b) Q of T-section if \( R_h = 5050 \) Ω (Note: \( Q_I > Q_L \))

L-1 section: \[ Q_1 = \sqrt{\frac{R_h}{R_1}} - 1 = \sqrt{\frac{5050}{50}} - 1 = 10 \]

L-2 section: \[ Q_2 = \sqrt{\frac{R_h}{R_2}} - 1 = \sqrt{\frac{5050}{600}} - 1 = 2.723 \]
T-Section: Design Example

Design a T-section LC matching network to match $R_1 = 50 \, \Omega$ and $R_2 = 600 \, \Omega$ at 400 MHz, and Q associated with $R_1$ is 10.

(a) For L-1 network, select $Q_1 = 10$.

(b) $R_h = R_1 (Q_1^2 + 1) = 50(10^2 + 1) = 5050 \, \Omega$

(c) $X_2' = \frac{R_h}{Q_1} = \frac{5050}{10} = 505 \, \Omega$  
   –ve for C (arbitrarily selected)

(d) $X_1 = R_1Q_1 = 50 \times 10 = 500 \, \Omega$  
   +ve for L (opposite of $X_2'$)
T-Section: Design Example

(a) For L-2 network,
\[ Q_2 = \sqrt{\frac{R_h}{R_2}} - 1 = \sqrt{\frac{5050}{600}} - 1 = 2.723 \]

(b) \[ X_2'' = \frac{R_h}{Q_2} = \frac{5050}{2.723} = 1854 \Omega \] +ve for L (arbitrarily selected)

(c) \[ X_3 = R_2Q_2 = 600 \times 2.723 = 1634 \Omega \] –ve for C (opposite of \( X_2'' \))

(d) \[ X_2 = X_2'//X_2'' = \frac{X_2' \times X_2''}{X_2' + X_2''} = \frac{-505 \times 1854}{-505 + 1854} = -694 \Omega \] –ve means C

(e) \[ X_1 \Rightarrow L_1 = \frac{X_1}{2\pi f} = \frac{500}{2\pi \times 400 \times 10^6} = 199 \text{ nH} \]

(f) \[ X_2 \Rightarrow C_2 = \frac{1}{2\pi f |X_2|} = \frac{1}{2\pi \times 400 \times 10^6 \times 694} = 0.57 \text{ pF} \]

(g) \[ X_3 \Rightarrow C_3 = \frac{1}{2\pi f |X_3|} = \frac{1}{2\pi \times 400 \times 10^6 \times 1634} = 0.24 \text{ pF} \]
T-Section: Design Example (cont'd)

$L_1 = 199 \, nH \quad C_2 = 0.57 \, pF \quad C_3 = 0.24 \, pF$

$50 \, \Omega \quad R_1 \quad jX_1 \quad L-1 \quad jX_2' \quad jX_2 \quad jX_2'' \quad L-2 \quad R_2 \quad 600 \, \Omega$

"AUTO-TRANSFORMER" another possible solution
**π-Section**

- **Advantages:**
  - $R_1$ can be $<$ or $>$ $R_2$
  - Variable Q (higher than L-section)
- **Disadvantage:**
  - More L C elements

**Method:**
(a) Introduce a hypothetical resistance level $R_h$ at $jX_2$, such that
\[
R_h < R_1 \quad \text{and} \quad R_h < R_2
\]
($R_h$ determines the Q of the matching network)
(b) Split $jX_2$ into $jX_2'$ and $jX_2''$ (in series)
(c) Treat the π-section as two L-sections (L-1 and L-2)

**NOTE:** Q of L-1 section > Q of 2-element L-section (as $R_h < R_2$)
π-Section: Design Example

Design a π-section LC matching network to match $R_1 = 50 \, \Omega$ and $R_2 = 600 \, \Omega$ at 400 MHz, and Q associated with $R_1$ is 10.

(a) For L-1 network, select $Q_L = 10$.

(b) $R_h = \frac{R_1}{Q_L^2 + 1} = \frac{50}{10^2 + 1} = 0.495 \, \Omega$

(c) $X_2' = R_h Q_1 = 0.495 \times 10 = 4.95 \, \Omega$ +ve for L (arbitrarily selected)

(d) $X_1 = \frac{R_1}{Q_1} = \frac{50}{10} = 5.0 \, \Omega$ –ve for C (opposite of $X_2'$)
\( \pi \)-Section: Design Example (cont'd)

(a) For L-2 network,
\[ Q_2 = \sqrt{\frac{R_2}{R_h}} - 1 = \sqrt{\frac{600}{0.495}} - 1 = 34.80 \]

(b) \( X_2 = R_h Q_2 = 0.495 \times 34.80 = 17.23 \Omega \) +ve for L (arbitrarily selected)

(c) \( X_3 = \frac{R_2}{Q_2} = \frac{600}{34.80} = 17.24 \Omega \) –ve for C (opposite of \( X_2'' \))

(d) \( X_2 = X_2' + X_2'' = 4.95 + 17.23 = 22.18 \Omega \) +ve means L

(e) \( X_1 \Rightarrow C_1 = \frac{1}{2\pi f |X_1|} = \frac{1}{2\pi \times 400 \times 10^6 \times 5} = 79.58 \text{ pF} \)

(f) \( X_2 \Rightarrow L_2 = \frac{X_2}{2\pi f} = \frac{22.18}{2\pi \times 400 \times 10^6} = 8.83 \text{ nH} \)

(g) \( X_3 \Rightarrow C_3 = \frac{1}{2\pi f |X_3|} = \frac{1}{2\pi \times 400 \times 10^6 \times 17.24} = 23.08 \text{ pF} \)
This is just one of the four possible solutions.
Common Transmission-Line Applications

- Inductor and capacitor
- Quarter-wave transformer
- Single-stub tuner
- Double-stub tuner
- Triple-stub tuner
Transmission Line Inductor

- The feed-point impedance of a terminated transmission line is:

\[ Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)} \]

- If the load impedance is a short-circuit, \( Z_L = 0 \), then

\[ Z_{in} = jZ_0 \tan(\beta l) = j\omega L \]

- Thus, the terminated transmission line behaves like an inductor (inductance = \( L \)) if \( bl < \pi/2 \).

\[ L = \frac{Z_0 \tan(\beta l)}{\omega} \]
Example

What is the equivalent inductance at 1 GHz at the feed-point of a 50 W, l/8 transmission line which is terminated with a short-circuit?

\[ L = \frac{Z_o \tan(\beta l)}{\omega} = \frac{50 \tan\left(\frac{2\pi \cdot \lambda}{l} \cdot \frac{\lambda}{8}\right)}{2\pi(10^9)} = 7.06\,nH \]
Transmission Line Capacitor

- The feed-point impedance of a terminated transmission line is:
  \[
  Z_{in} = Z_o \frac{Z_L + jZ_o \tan(\beta l)}{Z_o + jZ_L \tan(\beta l)}
  \]

- If the load impedance is an open-circuit, \(Z_L = \infty\), then
  \[
  Z_{in} = \frac{Z_o}{j \tan(\beta l)} = \frac{1}{j \omega C}
  \]

- Thus, the terminated transmission line behaves like a capacitor (capacitance = \(C\)) if \(bl < p/2\).
  \[
  C = \frac{\tan(\beta l)}{\omega Z_o}
  \]
Example

What is the equivalent capacitance at 1 GHz at the feed-point of a 50 W, l/8 transmission line which is terminated with an open-circuit?

\[ C = \frac{\tan(\beta l)}{\omega Z_o} = \frac{\tan\left(\frac{2\pi \cdot \frac{\lambda}{8}}{\lambda}\right)}{2\pi(10^9)50} = 3.18 \, \text{pF} \]
Quarter-Wave Transformer

- The feed-point impedance of a terminated transmission line is:
  \[ Z_{in} = Z_o \frac{Z_L + jZ_o \tan(\beta l)}{Z_o + jZ_L \tan(\beta l)} \]

- When \( l = \frac{\lambda}{4} \), i.e. \( bl = \frac{\pi}{2} \), then
  \[ Z_{in} = \frac{Z_o}{Z_L} \]

- Thus, we can transform the load impedance \( Z_L \) to another value \( Z_{in} \) by designing the \( \frac{l}{4} \) transmission line with a suitable characteristic impedance \( Z_o \):
  \[ Z_o = \sqrt{Z_{in}Z_L} = \sqrt{50 \times 75} = 61.2\Omega \]
Example

Match a 75 W load to a 50 W source at 300 MHz using a l/4 transformer.

The physical length of the transmission line is determined by the wavelength, which depends on the velocity factor of the transmission line.
STUB MATCHING
Single-Stub Tuner

- In a single-stub tuner, a short transmission line (called "stub") is added in parallel with the main transmission line.
- The stub usually has the same Zo as the main transmission line.
- By choosing the appropriate stub length d and its distance l from the load, it is possible to achieve zero reflection from the source to the point where the stub is added.
- Therefore, the stub is always placed close to the load.
- The stub is usually a short-circuit stub to minimize radiation loss (which is more likely to occur with an open-circuit stub).
Single-Stub Tuner

Source $Z_o$  

Short-circuit stub

Load $Z_L$

$l$, $d$
Single-Stub Tuner (cont'd)

• At point A where the stub is added, it is desired to have the impedance equal to \( Z_A = Z_{in} = Z_o \). It will be more convenient to use the normalized impedance \( z_A = z_{in} = 1 \).

• Also, as the stub is added in parallel, it is more convenient to work in admittances:

\[
\begin{align*}
y_o &= \frac{1}{z_o} = 1 \\
y_{in} &= \frac{1}{z_{in}} = 1 \\
y_L &= \frac{1}{z_L}
\end{align*}
\]

• The matching process involves:
  – using the distance \( l \) to transform \( y_L \) to \( 1 + jb \), and
  – using the stub to add \( -j_b \) to cancel the \( +j_b \) susceptance.

• Although the matching process can be carried out using equations, a Smith Chart is most conveniently used for this application.
Single-Stub Tuner Using the Smith Chart

- Locate $z_L$ and $y_L$. ($y_L$ is diagonally opposite $z_L$.)
- Using a compass, draw a locus towards the Generator along a constant $|\Gamma|$ circle until $r = 1$ circle is reached (Point A). The distance moved is $l$ (in units of $\lambda$).
- Read the $jb$ value at point A. (Two possible solutions.)
- The stub is required to have $-jb$.
- Choose either a short-circuit or open-circuit stub.
- Determine the stub length $d$ to produce the required susceptance.

**EXAMPLE: ($z_L = 2$)**
- Locate $z_L = 2$, and $y_L = 0.5$.
- Move towards Generator until point $P_A$ or $P_B$. Read $y_A = 1 + j0.71$ and $y_B = 1 - j0.71$
Antenna & Propagation

Impedance Matching

Short-circuit stub
Single-Stub Tuner Using the Smith Chart (cont’d)

EXAMPLE: \( z_L = 2 \)

- Locate \( z_L = 2 \), and \( y_L = 0.5 \).
- Move towards Generator until point \( P_A \) or \( P_B \).
- **Read on the circumference:** \( l_A = 0.152\lambda \) and \( l_B = 0.348\lambda \).
- Read at point \( P_A \) and \( P_B \):
  \[
  y_A = 1 + j0.71 \quad y_B = 1 - j0.71
  \]
- At point \( P_A \), the stub must have a susceptance of \(-j0.7\). If it is a short-circuit stub, its length \( d_{AS} = 0.402\lambda - 0.25\lambda = 0.152\lambda \). If it is an open-circuit stub, its length \( d_{AO} = 0.402\lambda \). (see next slide.)
- At point \( P_B \), the stub must have a susceptance of \(+j0.7\). If it is a short-circuit stub, its length \( d_{BS} = 0.098\lambda + 0.25\lambda = 0.348\lambda \). If it is an open-circuit stub, its length \( d_{BO} = 0.098\lambda \).
For a stub with a susceptance of $-j0.7$, if it is a short-circuit stub, its length $d_{AS} = 0.402\lambda - 0.25\lambda = 0.152\lambda$. If it is an open-circuit stub, its length $d_{AO} = 0.402\lambda$. 

**Single-Stub Tuner Using the Smith Chart (cont’d)**
Single-Stub Tuner: Exercise

The impedance of a WiFi monopole antenna at 2.48 GHz was 20 – j25 Ω. The antenna was connected to the transmitter via a 50 Ω microstrip line with an effective dielectric constant of 2. Design a single-stub matching network to match the antenna to the transmission line. Use the shortest short-circuit stub.
GAMMA MATCH
GAMMA MATCH

- Gamma Match
  - Unbalanced transmission lines.
  - Equivalent circuit
    - The gamma match is equivalent to half of the T-match
    - Requires a capacitor in series with the gamma rod
    - The input impedance is:

\[
Z_{in} = -jX_c + \frac{Z_g \left[ (1 + \alpha)^2 Z_a \right]}{2Z_g + (1 + \alpha)^2 Z_a}
\]

[3.21]

\(Z_a\) is the center point free space input impedance of the antenna in the absence of gamma-match connection.
GAMMA MATCH

• Design procedure
  - Determine the current division factor $\alpha$ by using Eq. [3.13]
  - Find the free space impedance (in the absence of the gamma match) of the driven element at the center point. Designate it as $Z_a$
  - Divide $Z_a$ by 2 and multiply by the step-up ratio $(1+\alpha)^2$. Designate the result as $Z_2$
  - Determine the characteristic impedance $Z_0$ of the transmission line form by the driven element and the gamma rod using Eq. [3.15a]

$$Z_2 = R_2 + jX_2 = (1 + \alpha)^2 \frac{Z_a}{2}$$ [3.22]
GAMMA MATCH

- Normalized $Z_2$ of Eq. [3.22] by $Z_0$ and designate it as $z_2$
- Invert $z_2$ of Eq. [3.23] and obtain its equivalent admittance $y_2 = g_2 + j b_2$
- The normalized value Eq. [3.24]

\[
\begin{align*}
  z_2 &= \frac{Z_2}{Z_0} = \frac{R_2 + j X_2}{Z_0} = r_2 + j x_2 \\
  z_g &= j \tan \left( k \frac{l'}{2} \right)
\end{align*}
\]
GAMMA MATCH

- Equivalent admittance \( y_g = g_g + j b_g \)
- Add two parallel admittances to obtain the total input admittance at the gamma feed.
- Invert the normalize input admittance \( y_{in} \) to obtain the equivalent normalized input impedance

\[
y_{in} = y_2 + y_g = (g_2 + g_g) + j (b_2 + b_g) \tag{3.25}
\]

\[
z_{in} = r_{in} + j x_{in} \tag{3.26}
\]
GAMMA MATCH

- Obtain the unnormalized input impedance by multiplying $z_{in}$ by $Z_0$
- Select the capacitor $C$ so that its reactance is equal in magnitude to $X_{in}$

$$Z_{in} = R_{in} + jX_{in} = Z_0 z_{in} \quad [3.27]$$

$$\frac{1}{\omega C} = \frac{1}{2\pi f C} = X_{in} \quad [3.28]$$
How they work
How they are made

BALUNS
What is a balun?

• A Balun is special type of transformer that performs two functions:
  – Impedance transformation
  – Balanced to unbalanced transformation

• The word balun is a contraction of “balanced to unbalanced transformer”
Why do we need a balun?

- Baluns are important because many types of antennas (dipoles, yagis, loops) are balanced loads, which are fed with an unbalanced transmission line (coax).
- Baluns are required for proper connection of parallel line to a transceiver with a 50 ohm unbalanced output.
- The antenna’s radiation pattern changes if the currents in the driven element of a balanced antenna are not equal and opposite.
- Baluns prevent unwanted RF currents from flowing in the “third” conductor of a coaxial cable.
Balanced vs Unbalanced Transmission Lines

- A balanced transmission line is one whose currents are symmetric with respect to ground so that all current flows through the transmission line and the load and none through ground.
- Note that line balance depends on the current through the line, not the voltage across the line.
An example of a Balanced Line

- Here is an example of a balanced line. DC rather than AC is used to simplify the analysis:

\[ V = +6 \text{ VDC} \]

\[ I = 25 \text{ mA} \]

\[ V = -6 \text{ VDC} \]

\[ I = -25 \text{ mA} \]

\[ 240 \Omega \]

- Notice that the currents are equal and opposite and the total current flowing through ground = 25mA-25mA = 0
FAQ’s

• Do I really need a balun?
  – Not necessarily. If you feed a balanced antenna with unbalanced line and you don’t want feed line radiation, use a balun!

• What kind of balun is best?
  – There is no best balun for all applications. The choice of balun depends on the type of antenna and the frequency range.

• Will you make a Balun for me?
  – No. However, I will be happy to show how to make your own.